

On the Thermal Conductivity of Hydrogen at Elevated Temperatures

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To evaluate the thermal conductivity of a hydrogen plasma at atmospheric pressure, previously reported results of experimental end-on spectroscopic temperature measurements are corrected for the influence of the schlieren effect. New thermal conductivity data for hydrogen are given.

I. Introduction

For temperatures above 7000 K there is a large discrepancy (by about a factor of 2) between calculated thermal conductivities of hydrogen plasmas¹ and corresponding results obtained on wall-stabilized arcs^{2,3}. It has been suggested that this discrepancy can be resolved by using a non-LTE approach to the problem⁴.

Recently it has been demonstrated that end-on plasma observations may be affected by the schlieren effect^{5,6}. Since the thermal conductivity depends strongly on the temperature, we report here an attempt to correct available spectroscopic measurements⁷. Corrected temperature profiles together with electrical measurements⁸ are further used to evaluate the transport properties of hydrogen at elevated temperatures.

II. Correction for Refractive-ray Bending

A detailed description of various numerical procedures to correct spectroscopic measurements is published elsewhere⁹, and therefore a minimum of details will be given here.

The equation of the ray trajectory through a circularly symmetric and optically thin plasma source with the x -axis coinciding with the plasma axis is given by the expression (see e.g. Reference 6).

$$\frac{dy}{dx} = \begin{cases} [(\mu_0/\mu)^2 - 1]^{1/2} & \text{for } d\mu/dy < 0, \\ -[(\mu/\mu_0)^2 - 1]^{1/2} & \text{for } d\mu/dy > 0 \end{cases} \quad (1)$$

where μ_0 is the refractive index at the point of incidence of the ray. The incident ray is assumed here to be parallel to the x -axis. The radius of curvature of the ray path defined by equation 1 is given by

$$R_0 = \mu^2/\mu_0 (d\mu/dy)_0. \quad (2)$$

For most laboratory plasmas the ratio of the plasma length, l , to the radius, R_0 satisfies the condition

$$l/R_0 = (d\mu/dy)_0/\mu_0 \ll 1 \quad (3)$$

and the deflection of the ray path, δ , may be calculated as follows:

$$\delta = R_0[1 - (1 - l^2/R_0^2)^{1/2}]. \quad (4)$$

This equation can be transformed into

$$\begin{aligned} \delta &= l^2/2 R_0 = l^2 (d\mu/dy)_0/2 \mu_0 \\ &= l^2 (d\mu/dT)_0 (dT/dy)_0/2 \mu_0. \end{aligned} \quad (5)$$

Thus, in calculating $\delta(T)$, the first step is to evaluate $\mu(T)$ for a hydrogen plasma at atmospheric pressure. The plasma refractive index is calculated at the wavelength of H_β where all temperature measurements were performed⁷. It should be noted that in evaluating $\mu(T)$ complete thermal equilibrium is assumed to be established. The contributions of excited species of hydrogen to the plasma refractive index were evaluated and taken into account. Data for mean electronic polarisabilities of hydrogen molecules and atoms were taken from Allen¹⁰. Plasma refractivity calculations showed that electrons play a dominant role at elevated temperatures ($> 13\,000$ K) and that the contribution of other plasma components may be neglected. This eliminates the questionable CLTE assumption. The electron density may be derived e.g. from the width of the H_β line or from interferometric measurements.

Refractive index data together with experimentally determined temperature profiles⁷ were used to calculate the trajectories of the rays [Eq. (5)], and these data were further used to obtain corrected temperature profiles. The finite aperture of the optical telecentric system used for end-on plasma observations⁷ had to be taken into account. This optical arrangement, frequently employed in plasma



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spectroscopy, allows only rays parallel to the axis (within the angle of the aperture) to reach the entrance slit of the spectrograph. It is customary to assume that, if a small aperture is used, a homogeneous plasma volume is observed. As will be shown later, refractive index gradients normal to the axis of observation may invalidate this assumption.

The boundaries of the plasma volume observed by a telecentric system are determined by its aperture and by the slit width of the spectrograph. They are drawn schematically in Fig. 1 with the lines

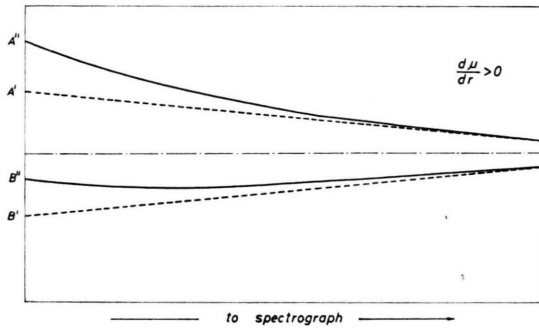


Fig. 1. Illustration of numerical procedure.

AA' and BB'. In the presence of refractive index gradients these boundary rays will be deflected (AA'' and BB'' in Fig. 1). In the case of a small aperture telecentric system, the deflection of the boundary rays can be evaluated from equation 5.

By evaluating trajectories of boundary rays in the plasma of a known temperature distribution, it is possible to calculate the intensity of radiation, I , which reaches the spectrograph at a particular wavelength. Therefore, to obtain corrected temperature profiles the following procedure is employed: first experimental profiles of $T(r)$ are used to evaluate the plasma emissivity, $\varepsilon(r)$, at the wavelength of H_β , and to calculate refractive index profiles $\mu(r)$. Secondly, the deflection of the boundary rays is evaluated from the relation

$$\delta = \frac{L}{2} \int_0^L \left[\frac{d\mu}{dr} \right]_{r=r(x)} dx. \quad (6)$$

In Fig. 2 three distortions for three radial positions in the arc are given. The corresponding areas are further used to calculate an average intensity, \bar{I}

$$\bar{I} = \frac{1}{\Delta - (\delta^+ - \delta^-)} \int_0^L \left[\int_{\delta^-}^{\delta^+} \varepsilon(r) dr \right] dx \quad (7)$$

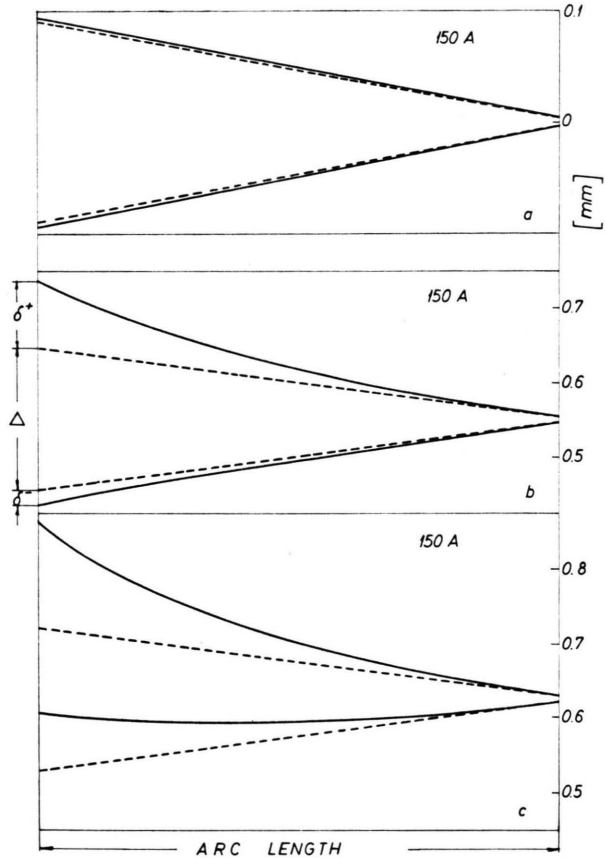


Fig. 2. Examples of possible distortions at various positions of the arc radius.

where Δ , δ^+ and δ^- are defined in Fig. 2 and L is the arc length. The average intensity, \bar{I} , is then related to the average radial coordinate, \bar{r} , determined by the relation

$$\bar{r} = r + \frac{1}{2}(\delta^+ + \delta^-). \quad (8)$$

The radial distribution of the electron temperature is deduced from the ratios of the H_β line intensity at various radial positions in the arc to the maximum intensity which occurs at 16 000 K at atmospheric pressure. In this way new temperature profiles were obtained and the whole procedure is repeated until convergence is achieved. It is to be noted that only a few iterations were necessary.

As to the importance of the schlieren effect in the end regions of the arc column where the discharge bends towards the electrodes, it was found that the light refraction in these regions is negligible for the wavelength of H_β , and in all calculations the arc column was treated as cylindrical without end regions.

III. Results and Discussion

Corrected and experimental temperature profiles⁷ are given in Figure 3. It is interesting to note that the corrected axial temperatures are systematically higher for arc currents larger than 60 A. With the corrections the agreement between the high-prec-

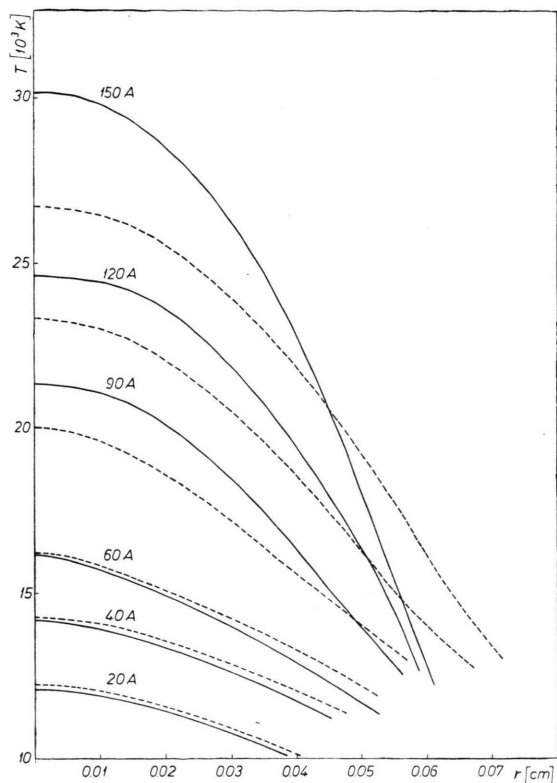


Fig. 3. Radial temperature distributions in a hydrogen arc. Dashed lines: experiment⁷; solid lines: corrected profiles.

sion hydrogen arc experiments by Steinberger⁷ and Behringer and Ott¹¹ is improved and is well within the limits of experimental errors. For example, for a 90 A arc the corrected axial temperature⁷ is 21 200 K while in reference¹¹ it is reported to be 21 100 K for the same arc current. This agreement is even more encouraging if one takes the difference of the methods into account, the axial temperature in Ref. 11 being deduced from end-on investigations of the hydrogen continuum in the near UV region of the spectrum.

It was also noticed that in another hydrogen arc experiment¹² (3 mm dia), where the spectroscopic plasma observations were performed side-on, for $T > 20\,000$ K the temperatures were always higher than in Reference 7. This discrepancy could now be explained, in great part, by the influence of the schlieren effect.

The corrected temperature profiles, Fig. 3, were used together with electrical measurements⁸ to derive the electrical conductivity, Fig. 4, as well as thermal conductivity of hydrogen, Figure 6. The numerical procedure described by Plantikow⁸ is employed for the evaluation of the thermal conductivity: first a corrected heat potential function is derived, Fig. 5, and then this is used to calculate the total thermal conductivity of hydrogen in the temperature range 9000 K–30 000 K, Figure 6. The agreement with theory¹ is improved especially in the region 10 000 K–20 000 K, but still a large discrepancy with the theory exists. The agreement with another experiment¹² is very good in the temperature range 10 000 K–20 000 K. However, for the discrepancy at higher temperatures one can not find a sound explanation.

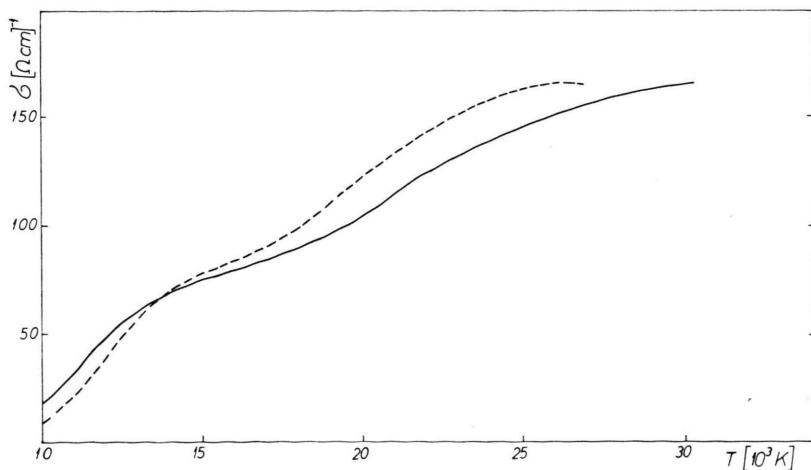


Fig. 4. Electrical conductivity of a hydrogen plasma at atmospheric pressure. Solid line: corrected curve.

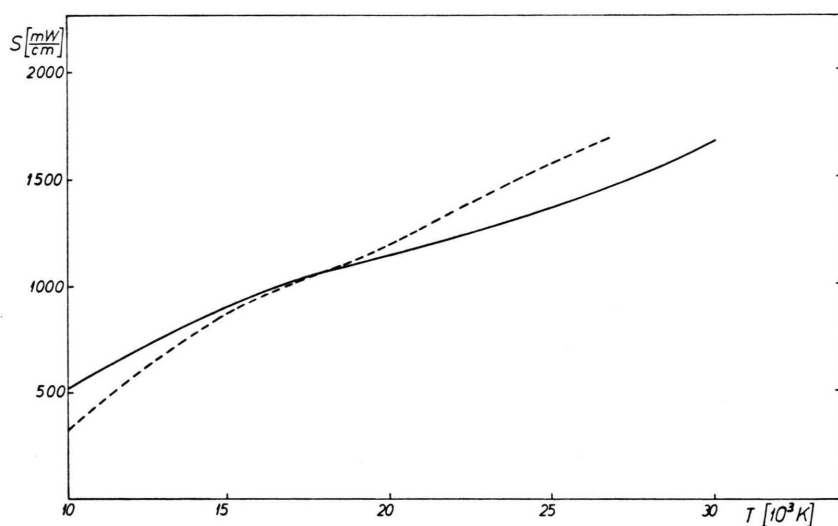


Fig. 5. Heat potential function of a hydrogen plasma at atmospheric pressure. Solid line: corrected curve.

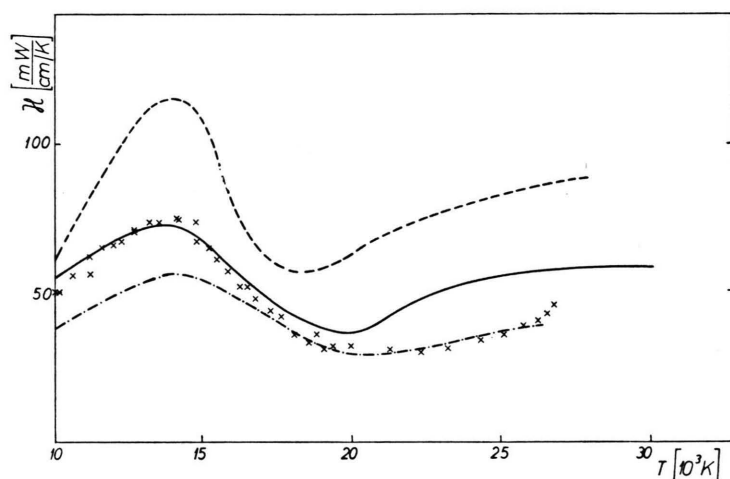


Fig. 6. Thermal conductivity of hydrogen at atmospheric pressure: experiment⁷ — — —; experiment¹² ×; present work — — —; theory¹ - · - · - .

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